20th
IRANIAN STUDENT CONFERENCE
ON ELECTRICAL ENGINEERING

Linear analysis for the first-harmonic based Colpitts QVCO

Mohammad Jafar Hemmati
Department of Electronic Engineering, Sirjan University of Technology, Sirjan, Iran
Mj.hemati@sirjantech.ac.ir

Abstract

In this work, the Colpitts LC oscillators are categorized into some groups to gain a good insight into the different types of Colpitts oscillators. This categorization helps the oscillator designers to choose a good Colpitts topology for designing of the specific circuit. In addition, the significant specifications of each category of Colpitts oscillators which should be carefully considered in the designing of the RF integrated circuits are explained. In fact, the architecture of oscillators significantly determines their specifications such as phase noise, power consumption, frequency tuning range, chip area, ease of implementation, immunity to circuit parasitic elements and ability to work at low voltages. Moreover, a recently published Colpitts quadrature voltage controlled oscillator (QVCO) realized by two Colpitts VCOs with the coupling scheme of in-phase anti-phase is theoretically analyzed. In fact, a linear analysis is presented to show the quadrature operation of the previously designed QVCO. Using this type of analysis, some other similarly first-harmonic QVCOs can be analyzed.

Keywords: Voltage controlled oscillator; quadrature; phase noise; current-switching;

1 Introduction

Low-noise low-power QVCOs are the indispensable block in many modern communication systems. In the recent years, there have been reported several major methods to generate quadrature signals (signals with 90^o phase differences): (1) poly-phase RC-CR filters, master–slave flip-flop used as a frequency divider, ring oscillator and coupling two identical VCO to each other. Among all these methods, coupling two identical LC-VCOs with a coupling circuit is a commonly used method for generating quadrature signals.

The cross-coupled and Colpitts LC oscillators are two popular topologies based on the negative resistance which are widely used in QVCOs. The cross-coupled topologies have the advantages of easy implantation and good oscillation start-up condition. Compared to the cross-coupled LC-VCO, the Colpitts oscillator exhibits better phase noise performance due to its superior cyclostationary noise properties (Aparicio, 2002). However, the Colpitts oscillators need to have higher start-up current for the oscillation because of their poorer start-up condition. Recently, the QVCO structures based on the Colpitts topologies have been widely studied due to their lower phase noise performances. Note that, to overcome the poor start-up condition of Colpitts oscillators, different g_m-boosting techniques have been adopted in the previously reported LC-VCOs (Cheng, 2013)(Ha, 2014)(Hong, 2009)(Hong, 2011)(Li, 2005).

In the next section, first, different types of the popular configurations of the LC Colpitts oscillators are categorized to give more insight into the LC oscillators. In this section, main specifications of the categorized Colpitts VCOs are pointed, too. In Section 3, a linear (sinusoidal) analysis is introduced to theoretically confirm the 90^o phase difference between the outputs waveforms of the previously reported QVCO. Note that, the following analysis provides a useful method to investigate the quadruple performance of the first-harmonic based QVCOs. Finally, the important points of this work are summarized in Section 4.

2 Categorization of the Colpitts oscillators

As mentioned above, Colpitts oscillator is one of the most popular topologies used in the QVCO structures. There Colpitts LC-VCOs can be divided into two main groups: single-ended and differential structures. Todays, single-ended Colpitts oscillators are rarely used in the integrated circuits because of their requiring to the higher start-up gain and their sensitivities to the parameter variations. Figure 1 shows three types of single-ended based topologies of Colpitts oscillators: gate-to-drain (GD), gate-to-source (GS) and drain-to-source (DS) capacitor feedback topologies (Razavi, 2017).

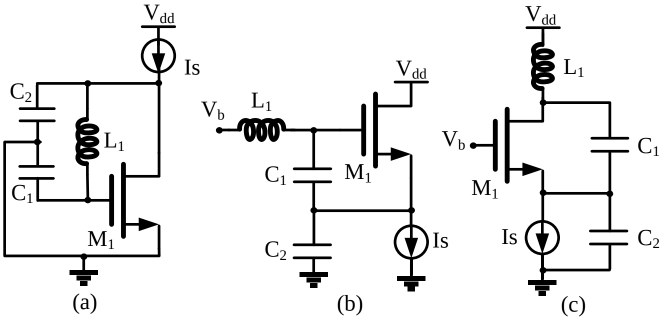


Figure. 1 Single-ended topologies of (a) GD, (b) GS and (c) DS Colpitts oscillator.

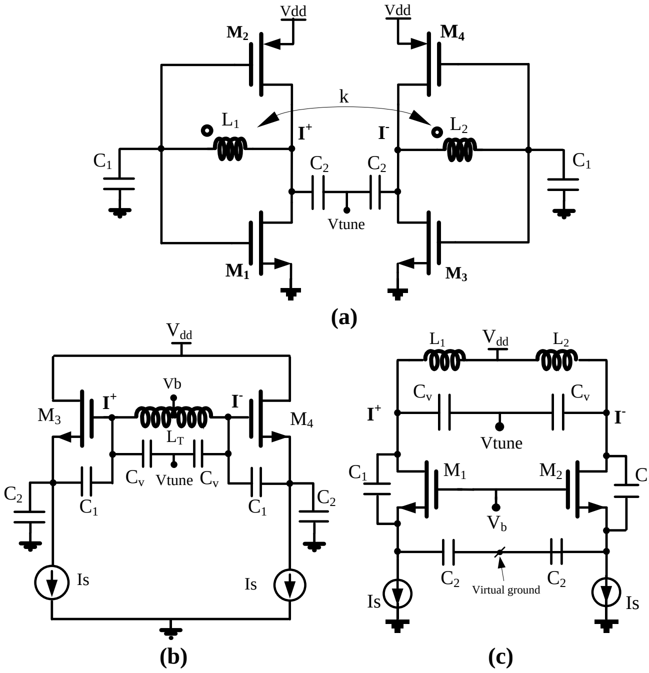
Compared to the single-ended structures, differential topologies are widely used in the RF integrated circuits such as QVCO and phase locked loops (PLLs). Generally, differential oscillators are being configured by coupling two identical single-ended ones. Hence, the popular reported configurations of differential Colpitts LC oscillators may be categorized into three groups. Figure 2 shows the conventional differential topologies of the GD, GS and DS capacitor feedback Colpitts oscillators. The conventional configuration of differential GD Colpitts oscillator presented in (Cha, 2008) has been shown in Figure 2 (a). This configuration consists of two single-ended GD Colpitts oscillator with the scheme of complementary. As can be seen, the two GD oscillators are differentially coupled together by the transformer coupling of L_1 and L_2. This structure has a better performance than the NMOS/PMOS-only structures. Additionally, it shows better up/down swing symmetry and higher transconductance (Cha, 2008).

The conventional configuration of the differential GS-Colpitts oscillator is shown in Figure 2 (b). This circuit oscillates differentially by coupling of the center-tapped inductor L_T (Hong, 2009). In Figure 2 (c) a differential DS Colpitts oscillator made by coupling two identical Colpitts oscillators and sharing their source-to-ground capacitors (C_2) is shown. Note that, the differential operation of this configuration will be guaranteed if the center node of capacitors C_2 is left floating and is not grounded (Aparicio, 2002). It should be noted that among differential Colpitts VCOs, the drain-to-source capacitor feedback based ones are the most popularly used oscillator types (Hong, 2011).

In the recent years, there have been introduced several differential structures of DS Colpitts VCOs realized by using the “current-reuse” and “current-switching” techniques. Figure 3 shows all these structures. Figure 3 (a) shows the schematic of a DS Colpitts VCO that is based on the current-reused technique proposed in (Yun, 2005). As explained in (Yun, 2005) (Lai, 2011), in this structure, the N/P type stacked transistors (M_1 and M_2) switch on and off at the same time. Hence, the current flows from supply voltage (V_{dd}) to ground at the half period only. In other words, this structure reduces power dissipation to half that of conventional Colpitts VCO.

In Figure 3 (b), a conventional configuration of the current-switching DS Colpitts VCO has been shown. This one is more commonly used topology of differential Colpitts VCOs. As can be seen, this configuration has been made up of two identical single-ended DS based Colpitts oscillators. In this configuration, the N-type switching transistors (M_{sw1} and M_{sw2}) alternately switch the current from one side to the other side of the oscillator and also provide a better phase noise performance due to the releasing of the trapped electrons. Note that, the current-switching Colpitts structures mainly have one cross-coupled pair of the transistor, at least. Also, the Colpitts VCO with the scheme of current-switching alleviates the noise perturbation at its output and achieves lower phase noise (Aparicio, 2002)(Razavi, 2017).

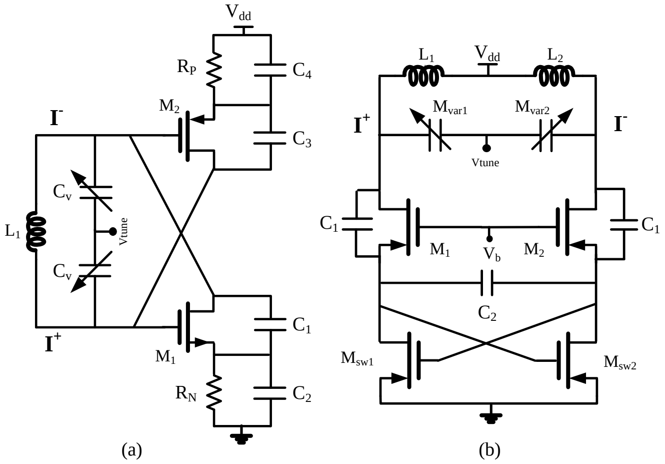
It should be noted that in the recently reported Colpitts LC-QVCOs, different coupling methods such as back-gate, transformer-based, capacitor-based and etc., have been proposed to couple two Colpitts LC oscillators. In some of them such as the QVCO in (Hemmati, 2014), the odd-harmonics (i.e. 1st, 3rd, ...) are used as the injecting signals for coupling from one core VCO to the other and in some other such as the circuit presented in (Gierkink, 2003), the even-harmonics (i.e. 2nd, 4th, …) are injected between two core VCOs. Hence, based on the mechanism of coupling, the Colpitts QVCOs can be divided into two main groups known as “first- harmonic” and “second-harmonic” (or superharmonic”). In the first-harmonic based QVCOs, the identical VCOs are coupled together in an "in-phase anti-phase" manner while in the QVCOs based on the synchronization of the second harmonic signals, a 180° phase difference is enforced between the signals of the common mode nodes (Gierkink, 2003).



(a)

(b) (c)

Figure. 2 Conventional differential Colpitts VCOs. (a) GD, (b) GS and (c) DS types.



(a) (b)

Figure. 3 Differential DS Colpitts VCOs. (a) Current-Reused, and (b) Current-Switched types

3 Analysis for the output quadrature waveforms

In the following, a linear analysis is presented to theoretically confirm the 90^{o} phase difference between the output voltages of the QVCO proposed in (Hemmati, 2014). The following analysis provides a good insight into the quadruple performance of the first-harmonic based QVCOs. In this analysis, it is assumed that the output waveforms of the QVCO shown in Figure 4 are sinusoidal. By applying the Kirchhoff's current law (KCL) to the nodes D_1 and D_3, the following equations are achieved. Note that, since no extra elements are used for the coupling in the circuit, the components appearing in the following relations are the capacitors C_2 and the parasitic capacitances (gate-drain, gate-source and gate-bulk) of the switching transistors.

As shown in Figure 5, the QVCO is simplified in two sub-circuit for more clarity of the analysis. It should be noted that the capacitor at the drain nodes of M_{1-4} have been considered with $2C_2$. In fact, in the QVCO circuit, C_2 is the equivalent of two shared capacitors with the values of $2C_2$. As shown in Figure 5, the gate-drain and gate-source parasitic capacitances of the switching transistors (M_{sw1-4}) are denoted with C_{gd-sw} and C_{gs-sw}, respectively. It should be noted that the gate-bulk capacitances (C_{gb-sw}) are not shown in this figure. In the following relations, V_1^+, V_1^-, V_2^+ and V_2^- refer to voltages at nodes D_1, D_2, D_3 and D_4, respectively. The KCL for the nodes V_1^+ in Figure 5 (a) and V_2^+ in Figure 5 (b) can be written as:

$$\begin{aligned}&I_1 - I_1^+ = V_1^+ 2C_2 j\omega + (V_1^+ - V_2^-)C_{gd-sw4} j\omega \\ &+V_1^+(C_{gs-sw4} + C_{gb-sw4}) j\omega\end{aligned} \tag{1}$$

$$\begin{aligned}&I_2 - I_2^+ = V_2^+ 2C_2 j\omega + (V_2^+ - V_1^+)C_{gd-sw1} j\omega \\ &+V_2^+(C_{gs-sw1} + C_{gb-sw1}) j\omega\end{aligned} \tag{2}$$

relations (1) and (2) rearranged as:

$$\begin{aligned}&I_1 - I_1^+ = V_1^+(2C_2 + C_{gd-sw4} + C_{gs-sw4} + C_{gb-sw4}) j\omega \\ &-V_2^- C_{gd-sw4} j\omega\end{aligned} \tag{3}$$

$$\begin{aligned}&I_2 - I_2^+ = V_2^+(2C_2 + C_{gd-sw1} + C_{gs-sw1} + C_{gb-sw1}) j\omega \\ &-V_1^+ C_{gd-sw1} j\omega\end{aligned} \tag{4}$$

assuming φ as the phase difference between the two identical oscillators, the voltages V_1^+, V_1^-, V_2^+ and V_2^- can be replaced by A, -A, $Ae^{j\varphi}$, $-Ae^{j\varphi}$ in phasor domain where A is the amplitude of each potential. Since the two symmetric oscillators are identical, the amplitude and frequency of each signal at the same nodes and branches of VCOs are equal. Hence, the relation between them can be written as:

$$I_2 - I_2^+ = e^{j\varphi}(I_1 - I_1^+) \tag{5}$$

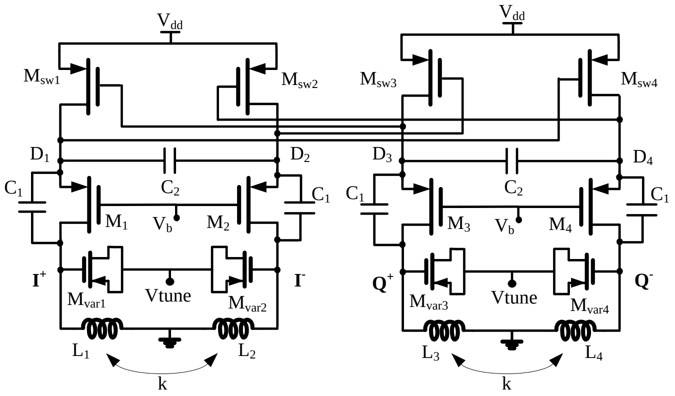
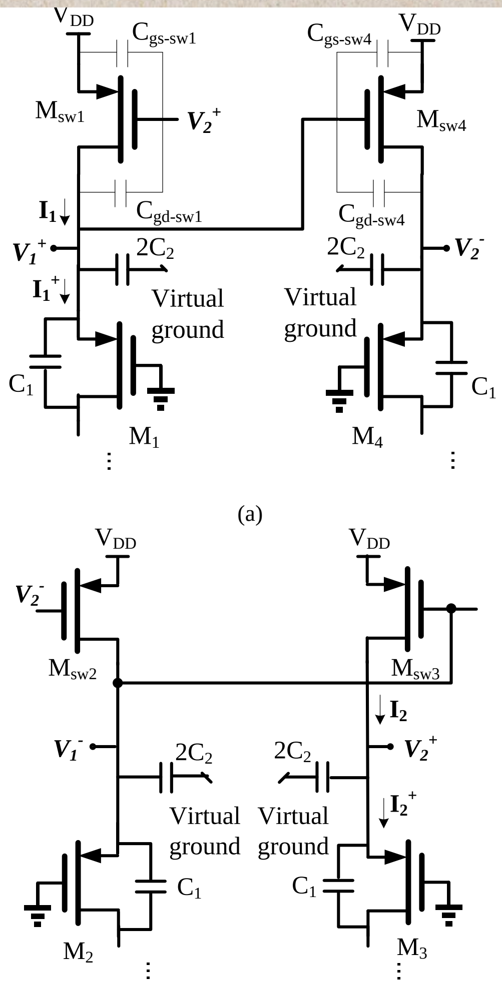


Figure. 4 Schematic of the current-switched Colpitts QVCO proposed in (Hemmati, 2014)



(a)

(b)

Figure. 5 Simplified schematic of the proposed QVCO in (a) Left-half of the first VCO and right-half of the second VCO with the connection between them (b) Reversed case of (a)

Therefore, using (3)-(5) the following equation is obtained:

$$V_2^+(2C_2+C_{gd-sw1}+C_{gs-sw1}+C_{gb-sw1})j\omega-V_1^+C_{gd-sw1}j\omega= \\ e^{j\varphi}[V_1^+(2C_2+C_{gd-sw4}+C_{gs-sw4}+C_{gb-sw4})j\omega-V_2^-C_{gd-sw4}j\omega] \qquad (6)$$

Since the core VCOs have the same switching transistors, the parasitic capacitances of them are equal (i.e. $C_{gd-sw1}= C_{gd-sw4}$). Hence, by replacing $2C_2+C_{gd-sw1}+C_{gs-sw1}+C_{gb-sw1}$ and $2C_2+C_{gd-sw4}+C_{gs-sw4}+C_{gb-sw4}$ with C_p and considering the voltages V_1^+, V_2^+ and V_2^- in phasor domain, relation (7) can be obtained as follows:

$$Ae^{j\varphi}C_pj\omega-AC_{gd-sw1}j\omega=e^{j\varphi}[AC_pj\omega-(-Ae^{j\varphi})C_{gd-sw4}j\omega] \qquad (7)$$

manipulating (7) gives:

$$AC_{gd-sw1}+Ae^{j2\varphi}C_{gd-sw4}=0\Rightarrow e^{-j\varphi}+e^{j\varphi}=0 \qquad (8)$$

and therefore:

$$2\cos\varphi = 0 \Rightarrow \varphi = k\pi + \pi/2 \qquad k = 0,1,2,\ldots \tag{9}$$

which indicates the voltages at nodes D_1 to D_4 ($V_1^{\pm}$ and $V_2^{\pm}$), and also outputs (I^+, I^-, Q^+ and Q^-) are in quadrature. Note that, although the above analysis is based on the sinusoidal waveforms, numerous simulations done in (Hemmati, 2014) show that the outputs are in quadrature even for non-sinusoidal waveforms.

4 Conclusion

Beside the cross-coupled structures, Colpitts oscillators are popularly used for generating the quadrature signals due to their low phase noise performances. In this work, several topologies of the single-ended and differential Colpitts LC oscillators were categorized to give a good insight into the Colpitts VCO structures. In fact, this work helps to choose a good VCO structure for the designing in specific applications. A linear analysis was presented to confirm the quadrature outputs of the previously designed QVCO. In the analysis, the presented equations have been written by applying KCL at two main nodes of the identical VCOs. This type of analysis can be applied to some other similarly first-harmonic QVCOs to show the output waveforms are in quadrature.

References

[1] Aparicio R, Hajimiri A (2002) A noise-shifting differential Colpitts VCO. IEEE journal of solid-state circuits. Vol. 37. No. 12. 1728-36

[2] Ebrahimi, E., & Naseh, S. (2012). A Colpitts CMOS quadrature VCO using direct connection of substrates for coupling. IEEE transactions on very large scale integration (VLSI) systems, Vol. 21. No. 3. 571-574

[3] Cha, C.-Y., & Lee, S.-G. (2005). A complementary Colpitts oscillator in CMOS technology. IEEE transactions on microwave theory and techniques, Vol. 53. No. 3, 881-887

[4] Cha, C.-Y. (2008). A source coupled differential CMOS complementary Colpitts oscillator with on-chip transformer tank. IEEE transactions on microwave theory and techniques, Vol. 56. No. 5. 1076-1082

[5] Cheng, K.-W., & Je, M. (2013). A Current-Switching and Gm-Enhanced Colpitts Quadrature VCO. IEEE Microwave and Wireless Components Letters, Vol. 23. No. 3, 143-145

[6] Gierkink, S. L., Levantino, S., Frye, R. C., Samori, C., & Boccuzzi, V. (2003). A low-phase-noise 5-GHz CMOS quadrature VCO using superharmonic coupling. IEEE journal of solid-state circuits, Vol. 38. No. 7, 1148-1154.

[7] Ha, K.-W., Ryu, H., Lee, J.-H., Kim, J.-G., & Baek, D. (2014). Gm-Boosted Complementary Current-Reuse Colpitts VCO With Low Power and Low Phase Noise. IEEE Microwave and Wireless Components Letters, Vol. 24. No. 6, 418-420

[8] Hemmati, M. (2014). Ultra-low-phase-noise CMOS LC quadrature voltage controlled oscillator with Colpitts topology. Electronics letters, Vol. 50. No. 3, 166-168

[9] Hong, J.-P., & Lee, S.-G. (2009). Low Phase Noise Gm-Boosted Differential Gate-to-Source Feedback Colpitts CMOS VCO. IEEE journal of solid-state circuits, Vol. 44. No. 11. 3079-3091

[10] Hong, J.-P., & Lee, S.-G. (2011). Gm-boosted differential drain-to-source feedback Colpitts CMOS VCO. IEEE transactions on microwave theory and techniques, Vol. 59. No. 7, 1811-1821

[11] Lai, H.-H., Shen, I.-S., & Jou, C. F. (2011). Colpitts current-reused QVCO based on capacitor coupling. Paper presented at the Asia-Pacific Microwave Conference 2011

[12] Li, X., Shekhar, S., & Allstot, D. J. (2005). Gm-boosted common-gate LNA and differential Colpitts VCO/QVCO in 0.18 µm CMOS. IEEE journal of solid-state circuits, Vol. 40. No. 12. 2609-2619

[13] Razavi, B. (2017). Design of analog CMOS integrated circuits: Tata McGraw-Hill Education. Second Edition.

[14] Yun, S.-J., Shin, S.-B., Choi, H.-C., & Lee, S.-G. (2005). A 1mW current-reuse CMOS differential LC-VCO with low phase noise. Paper presented at the ISSCC. 2005 IEEE International Digest of Technical Papers. Solid-State Circuits Conference, 2005.